TABLE II.

Pressure in millims.	832·7	695·3	551·5	498·1	468·6
Temperature C.	11·1	11·0	10·45	11·1	11·1
Absorption coefficient of $Mg\,SO_4$ solution	1·2467	0·9331	0·8823	0·8974	0·8221
Absorption coefficient of water	1·3052	1·0445	0·8461	0·7546	0·7014

TABLE III.

Pressure in millims.	554·9	683·8	765·3	770·8	805·2	869·5
Temperature C.	10·1	12·9	13·3	11·1	11·1	11·65
Absorption coefficient of $Ca\,SO_4$ solution	0·8845	0·9923	1·0651	1·1885	1·2191	1·2964
Absorption coefficient of water	0·8617	0·9618	1·0624	1·1534	1·2048	1·2757

The general result of these experiments is, that sulphate-of-lime solution absorbs a little more carbonic acid than water, but follows the same law of variation with temperature and pressure; sulphate-of-magnesia solution differs slightly from water when but little time is left for the reaction to complete itself. If, however, the gas and solution are left in contact for a considerable time, the difference between the coefficients of water and of the salt solution becomes very marked, that of the latter being less for high pressures and greater for low ones than that of water.

The details of these experiments will be found elsewhere in a more extended paper.

II. "On an Instrument for the Composition of two Harmonic Curves." By A. E. DONKIN, M.A., F.R.A.S., Fellow of Exeter College, Oxford. Communicated by W. SPOTTISWOODE, Treas. R.S. Received November 6, 1873.

The interest in such compound curves lies in the fact that as a simple harmonic curve may be considered to be the curve of pressure on the tympanic membrane when the ear is in the neighbourhood of a vibrating body producing a simple tone, so a curve compounded of two such simple harmonic curves will be the curve of pressure for the consonance of the two tones which they severally represent, and thus the effect on the ear of different consonances can be distinctly represented to the eye.

If the motion of a point be compounded of rectilinear harmonic vibrations and of uniform motion in a straight line at right angles to the direction of those vibrations, the point will describe a simple harmonic curve.

Thus a pencil-point performing such vibrations upon a sheet of paper moving uniformly at right angles to their direction would draw such a curve.

The same kind of curve would also be drawn by keeping the pencil fixed and by giving to the paper, in addition to its continuous transverse motion, a vibratory motion similar and parallel to that which the pencil had; and if the motion of the latter be now restored, a complicated curve will be produced whose form will depend on the ratio of the numbers of

vibrations in a given time of the pencil and paper, and which will be the curve of pressure for the interval corresponding to this ratio.

The manner in which these three motions are combined in the machine is as follows:—Two vertical spindles, A and B, revolving in a horizontal

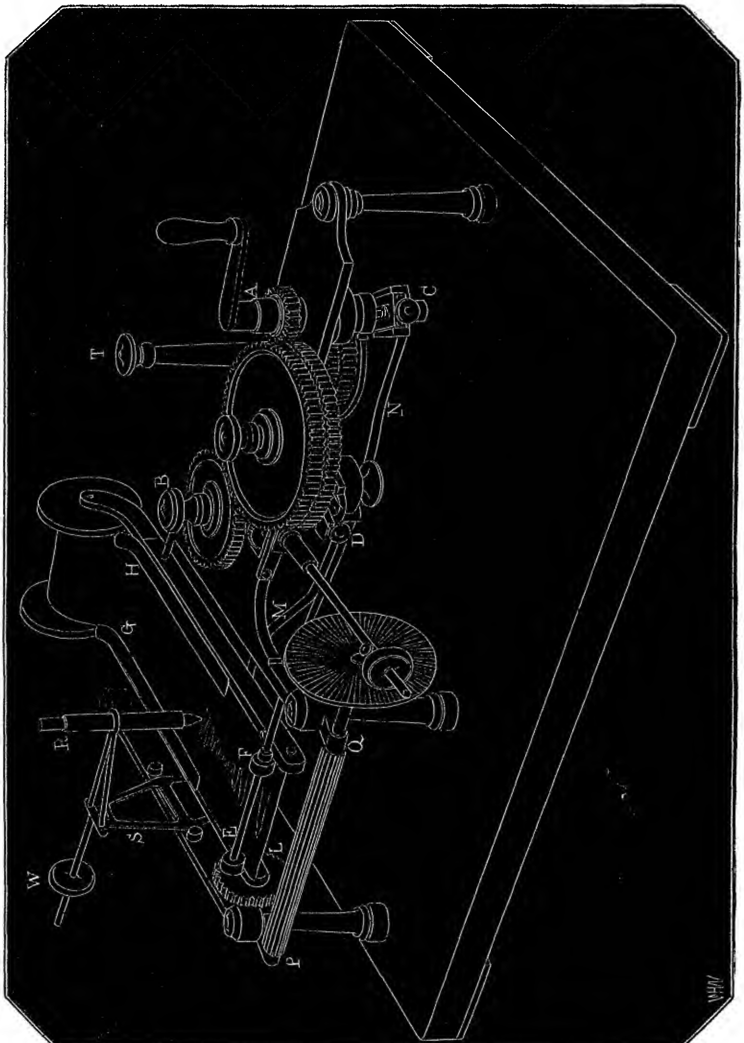

plate carry at their lower ends each a crank, C and D, and at their upper ends each a wheel cut with a certain number of teeth; these two wheels can be connected by means of an intermediate one, as is seen in the figure; and since either wheel of the pair can be replaced by another with a different

number of teeth, the relative angular velocities of the spindles can be regulated at pleasure. The paper upon which the curve is to be drawn is carried upon a rectangular frame, E F G H, capable of sliding horizontally up and down in a direction parallel to that of the plane passing through the spindles. This frame has a pair of rollers, E F and G H, at each end connected by tape-bands, between which the paper passes as the rollers turn. In order to give a motion of revolution to the rollers, a wheel, L, is fixed upon the axis of one of them whose teeth gear into those of a pinion, P Q, alongside which the frame slides, and which is itself driven by one of the vertical spindles. A connecting-rod, D M, is carried to the frame from the crank of this spindle, so that upon turning the latter a vibratory motion is given to the former; and since the transverse motion of the paper also depends upon the same spindle, a fixed pencil-point resting on it would draw a simple harmonic curve whose amplitude would depend on the radius of the crank, and wave-length on the transverse speed of the paper, which can be regulated at pleasure by means contrived for the purpose*.

A vibratory motion similar and parallel to that of the frame is given to a small tubular glass pen, R, so arranged as to move with its point lightly resting upon the paper. This motion is communicated by a connecting-rod, C N, from the other crank, which is carried underneath the sliding-frame and jointed to the lower end of a small vertical lever, S, to whose upper end the arm carrying the pen is attached.

The weight W serves to regulate the pressure of the pen on the paper, as it can be screwed in or out. T is merely a pillar upon which the change-wheels can be placed for convenience.

If the pair of wheels on the spindles are now connected by the intermediate one, it is plain that, upon turning either of the spindles by a winch provided for the purpose, the two motions of the paper will be combined with that of the pen, and the curve drawn will be that composed of the two simple harmonic ones, which would be the result of separately combining the harmonic vibrations due to each crank with the transverse motion of the paper. Thus if m and n are the numbers of teeth on the pair of wheels respectively, the equation to the resultant curve will be

$$y = \sin mx + \sin nx.$$

This equation implies that not only are the radii of the cranks the same, but also that they start parallel to each other and at right angles to the vertical plane passing through their axes: both these conditions can, however, be altered; and therefore the general form of equation to the curves which the machine can draw will be

$$y = a \sin (mx + \alpha) + b \sin (nx + \beta),$$

* It should be observed here that the vibratory motion thus given to the frame is not truly harmonic. In order to make it so, a more complicated contrivance than the simple crank and connecting-rod would have to be adopted; but this would probably introduce, through unavoidable play, an error greater than the present one, the length of the connecting-rods and the small size of the cranks rendering the latter nearly inappreciable. The motion will, however, for the sake of convenience, be considered truly harmonic throughout

where a and b are the radii of the cranks, and α and β are dependent on their relative inclinations to the above-mentioned vertical plane at starting.

As an example, suppose that $a=b$, while the ratio of m to n is as 2 to 1; then the above equation will represent the curve of pressure for the octave. Similarly, if m is to n as 16 to 15, the resultant curve represents the effect on the ear of a diatonic semitone, while the ratio 81 to 80 would give that of the comma. In both these curves, and more especially in the latter, the beats which would ensue on actually sounding the two tones together are shown with remarkable distinctness.

As the machine is provided with a set of change-wheels, many different curves can be produced, while the form of each can be more or less changed by altering the relative positions of the cranks before bringing the idle wheel into gear. It is also possible to obtain very large values of m and n in the above equation by using two idle wheels on the same axis which shall come into gear, the upper one with the wheel on the one spindle, the lower one with that on the other.

Thus, suppose A and B are the numbers of teeth on the spindle-wheels respectively, C and D those on the idle wheels, and let A gear with C and D with B; then $\frac{m}{n}=\frac{\mathrm{BC}}{\mathrm{AD}}$. Now, by properly choosing the four wheels, large values of m and n may be obtained. If, for instance, $\mathrm{A}=81$, $\mathrm{B}=80$, $\mathrm{C}=55$, and $\mathrm{D}=27$, $\frac{m}{n}=\frac{4400}{2187}$, this ratio being nearly $=\frac{2}{1}$, the corresponding curve will represent the effect of an octave slightly out of tune. The period of such curves as these being very long, it is necessary to have a good supply of paper; and this is arranged by carrying a reel-full on the horizontal frame, from which it is slowly unwound between the rollers. The rate at which this takes place has a good deal of influence on the form of the resultant curve; the slower it is the more compressed will the latter appear. Instead of using paper, the curves, provided the periods are short enough, may be drawn on slips of blackened glass, which can be carried along between the tapes connecting the rollers; they can be at once placed in a lantern and thrown on a screen.

The width of contour of any curve depends on the radii of the cranks; these may have any value between 0 and half an inch, and therefore the limit of possible width at any part will be two inches; so also, by altering the radii, a series of curves may be produced corresponding to the consonances of tones not of the same intensities. Since the maximum width of any curve will be double the sum of the radii of the cranks, the paper is cut to a width of two and a half inches, within which all curves which can possibly be drawn will be comprised.

The instrument is constructed by Messrs. Tisley and Spiller, of Brompton Road, to whom some improvement upon the original model is due.